

Problem 1 – Independence. Suppose we pull two cards from a standard deck of cards (with 52 cards, 26 black cards, and 4 Aces). Let X be the number of Aces and Y be the number of black cards in our two cards.

- (a) How many possible outcomes are there when pulling two cards (at the same time) from the deck?
- (b) Compute $\Pr(X = 2)$.
- (c) Compute $\Pr(Y = 2)$.
- (d) Compute $\Pr(X = 2, Y = 2)$.
- (e) Are X and Y independent?

Problem 2 – Indicators. Suppose we pull 5 cards from a standard 52-card deck containing 4 Aces. Let X be the number of Aces in my hand. We'll compute $E(X)$ two ways.

- (a) What is the probability that the first card I pull is an Ace?

- (b) What is the probability that the second card I pull is an Ace? Note that we have two conditions: when the first card I pull is an Ace, and when the first card I pull is not an Ace.

- (c) Use a counting argument to show that the probability that the i th card (for $1 \leq i \leq 5$) is an Ace is $\frac{1}{13}$.

- (d) Compute $E(X)$ using indicator variables, where $I_i = 1$ if an Ace is the i th card in my hand (for $1 \leq i \leq 5$).

- (e) Now we'll work towards a different indicator variable. What is the probability that the Ace of Spades is in my 5-card hand?

- (f) Compute $E(X)$ using indicator variables, where $I_i = 1$ if the i th Ace is in my hand (for $1 \leq i \leq 4$).