Problem 1 Use the stars and bars technique to count the number of outcomes in the following scenarios:

(a) How many ways are there to order 10 scoops of ice cream from 5 flavors?

(b) How many ways are there to place 10 indistinguishable balls in 5 bins?

(c) From a bin with 5 distinguishable balls, you draw a ball 10 times, with replacement. How many possibilities are there for the number of times each ball is drawn?

(d) How many solutions are there to the equation $a + b + c + d + e = 10$ where $a, b, c, d, e \geq 0$?

Problem 2 Each of the following problems requires at least one of the tricks from class: PIE, Complementary Counting, and Stars and Bars. Some use multiple!

(a) How many positive integers less than 100 are a multiple of 3 or 5?

(b) How many positive integers less than 100 are not a multiple of 3 or 5?

(c) A painter has been commissioned to paint 5 houses. The paint shop has three different colors to choose from, and the painter buys a bucket of paint per house. How many different paint orders could be made? (Two paint orders are considered the same if they have the same amount of each color of paint, regardless of order)

(d) A painter has been commissioned to paint 5 houses. They have three different colors to choose from, but at least one pair of adjacent houses must have the same color. How many ways are there to paint the houses?

(e) Ten distinguishable trees need to be planted in a row. However, if the tallest tree and the shortest tree are next to one another, the shortest tree will not get any sunlight. How many ways are there to plant the trees so that the tallest tree and shortest tree are not adjacent?

(f) Ten indistinguishable trees need to be planted along a row spanning three blocks. How many ways are there to distribute the trees among the blocks?

Problem 3 Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter, three-digit arrangement is equally likely, find the probability that such a license plate will contain at least one palindrome (contains a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left, or both).

For example, each of these license plates contain a palindrome: ABA123, ABC121, ABA121.

(a) Use complementary counting (compute the number of license plates that do *not* contain a palindrome, and subtract this from the total number of license plates).

(b) Use PIE (let A be the set of license plates with letter-based palindromes and B be the set of license plates with number-based palindromes)