

Problem 1 Your local ATM only dispenses 2 and 5 dollar bills, but the farmers market requires exact change! Prove that for any $n \geq 4$, n dollars can be made up of 2 and 5 dollar bills.

Problem 2 The chicken nugget problem is a real math problem, originally posed when McNuggets came in packs of 6, 9, and 20 in the UK. Show that for every $n \geq 44$, we can get n chicken nuggets from packs of 6, 9, and 20 nuggets.

Problem 3 I claim that I can show, by strong induction, that for every $n \geq 0$, $5n = 0$.

Find the error in my proof:

Base case: Let $n = 0$. Then $5 \cdot 0 = 0$, so our base case holds.

Inductive hypothesis: Let $k \in \mathbb{N}$ be arbitrary. Assume that $5j = 0$ for all $0 \leq j \leq k$.

Inductive step: We show that $5(k+1) = 0$. Note that $k+1 = i+j$ for some $i, j < k+1$. Therefore $5(k+1) = 5(i+j) = 5i + 5j = 0 + 0 = 0$ (by our inductive hypothesis). Therefore, for every $n \geq 0$, $5n = 0$. $\square$