**Problem 1 – Grab Bag.** This is a problem to get some practice and familiarity with the topics from number theory. Full proofs are not necessary.

(a) For what numbers $a$ does $-1|a$?

(b) For what numbers $a$ does $a| - 1$?

(c) True or False: for positive integers $a$ and $b$, if $a|b$ then $a \leq b$.

(d) Find $q$ and $r$ satisfying the division theorem for $n = -15$, $d = 7$.

(e) Use Euclid's algorithm to find $\gcd(483, 91)$.

(f) Write out the elements of the set $S = \{p + n : \exists p \in \mathbb{N}, \exists n \in \mathbb{N}, n = p^2, n < 10\}$.

**Problem 2 – GPT4 vs. Number Theory.** In this problem, I have asked GPT4 to prove a number theory statement. Identify everywhere a mistake is present in the solution.

**Problem:** Prove that if 27 divides $XYZ$, then 27 also divides $YZX$. (Here, $XYZ$ is the three-digit number with **digits** $X$, $Y$, and $Z$, not $X \times Y \times Z$.)

**GPT4 Solution:**

(1) To prove this, we will use the fact that a number is divisible by 27 if and only if the sum of its digits is divisible by 27.

(2) Let the three-digit number $XYZ$ be represented as $100X + 10Y + Z$. Here, $X$, $Y$, and $Z$ are the digits of the number and each ranges from 0 to 9. We are given that 27 divides $XYZ$, i.e., 27 divides $(100X + 10Y + Z)$.

(3) Consider the number $YZX$, which can be represented as $100Y + 10Z + X$. We need to show that 27 also divides $(100Y + 10Z + X)$.

(4) We start by rearranging the expression for $XYZ$:

$$100X + 10Y + Z = 99X + X + 10Y + Z = 99X + (X + 10Y + Z)$$

(5) Since 27 divides $(100X + 10Y + Z)$, it must also divide $(X + 10Y + Z)$ because $99X$ is a multiple of 27 (as $99 = 3 * 33$, and 27 is a multiple of 3).

(6) Now, consider the expression for $YZX$:

$$100Y + 10Z + X = 99Y + Y + 10Z + X = 99Y + (Y + 10Z + X).$$

(7) We already know that 27 divides $(X + 10Y + Z)$. We can rearrange $(X + 10Y + Z)$ as follows:

$$X + 10Y + Z = X + 10(Y + Z) - 9Z = X - 9Z + 10(Y + Z).$$

(8) Similarly, rearranging $(Y + 10Z + X)$:

$$Y + 10Z + X = Y + 10(Z + X) - 9X = Y - 9X + 10(Z + X).$$

(9) Since $(X + 10Y + Z)$ and $(Y + 10Z + X)$ are cyclic permutations of the digits, they are congruent modulo 9 (as subtracting 9 times a digit does not affect divisibility by 9). Therefore, if 27 divides one of these sums, it divides both sums.

(10) Thus, since 27 divides $(X + 10Y + Z)$, it must also divide $(Y + 10Z + X)$. Therefore, 27 divides $100Y + 10Z + X$, which means 27 divides the number $YZX$.

(11) This concludes the proof that if 27 divides a three-digit number $XYZ$, then 27 also divides $YZX$.

**Problem 3** Now it's your turn! Prove that if 27 divides $XYZ$, then 27 also divides $YZX$. *Hint: GPT-4 did do some things right! In particular,* $XYZ = 100X + 10Y + Z$ *is a helpful place to start.*